S.N.J.B.'s K.K.H.A. Arts, S.M.G.L. Commerce & S.P.H.J. Science College, Chandwad

Department of Mathematics

Paper II MT-212(A) Discrete Mathematics

# Practical 1

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**Q.1: Express compound propositions as English statements.**

i. Let p and q be the propositions "The election is decided" and "The vote have been counted" respectively.

a) $\neg p \wedge q$ b) $q \longrightarrow p$ c) $\neg q \longrightarrow \neg p$

d) $\neg p \longrightarrow \neg q$ e) $p \leftrightarrow q$ f) $\neg q \vee (\neg p \wedge q)$

ii. Let p, q and r be the propositions
p: You have the flu.
q: You miss the final examination
r: You pass the course.

a) $p \longrightarrow q$ b)$\neg q \leftrightarrow r$ c)$q \longrightarrow \neg r$

d) $p \vee q \vee r$ e) $(p \longrightarrow \neg r) \vee (q \longrightarrow \neg r)$ f) $(p \wedge q) \vee (\neg q \wedge r)$

**Q.2: Construct truth table for the following compound statements.**

i. $(p \longrightarrow q) \longrightarrow (q \longrightarrow p)$
ii. $p \longrightarrow (\neg q \vee r)$
iii. $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
iv. $(p \vee q) \wedge \neg r$

**Q.3: Determine whether given expression is a tautology.**

i. $[(p \longrightarrow q) \wedge (q \longrightarrow r)] \longrightarrow (p \longrightarrow r)$
ii. $[(p \longrightarrow q) \longrightarrow r] \leftrightarrow [p \longrightarrow (q \longrightarrow r)]$

**Q.4: Problems on Logical Equivalences.**

i. Show that $\neg(p \leftrightarrow q) and\ \neg p \leftrightarrow q$ are logically equivalent.
ii. Show that $(p \wedge q) \longrightarrow r\ and\ (p \longrightarrow r) \wedge (q \longrightarrow r)$ are not logically equivalent.

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## Practical 2

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1. (i) Determine the truth value of each of given statement

(a)- $\forall\, x\,(x^2 > x)$ (b)- $\forall\, x\,(x > 0 \lor x < 0)$

(c) $\forall x\,(x = 1)$ (d) $\exists\, x\,(x^2 = 2)$

Where the domain consists of all integers.

(ii) Determine the truth value of each of these statements if the domain consists of all integers

(a) $\forall\, n(n + 1 > n)$ (b) $\exists n\,(2n = 3n)$

(c) $\exists n\,(n = -n)$ (d) $\forall n\,(n^2 \geq n)$

(iii) Determine the truth value of each of these statements if the domain consists of all real numbers.

(a) $\exists x(x^3 = -1)$ (b) $\exists x(x^4 < x^2)$

(c) $\forall x((-x)^2 = x^2$ (d) $\forall x(2x > x)$

(iv) Determine the truth value of each of these statements if the domain for all variables consist of all integers.

(a)- $\forall n(n^2 \geq 0)$ (b) $\exists n(n^2 = 2)$

(c) $\forall n(n^2 \geq n$ (d) $\exists n(n^2 < 0)$

(v) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

(a) $\exists x(x^2 = 2)$ (b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 2 \geq 1$ (d) $\forall x(x^2 \neq x)$

(vi) Determine the truth value of eachof these statements if the domain of each variable consist of all real numbers.

(a) $\forall x \exists y(x^2 = y)$ (b) $\forall x \exists y(x = y^2)$

(c) $\exists x \forall y(xy = 0$ (d) $\exists x \exists y(x + y \neq y + x)$

(e) $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$ (f) $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$

(g) $\forall x \exists y(x + y = 1)$ (h) $\exists x \exists y(x + 2y = 2 \land 2x + 4y = 5)$

(i) $\forall x \exists y(x + y = 2 \land 2x - y = 1)$ (j) $\forall x \forall y \exists z(z = (x + y)/2)$

2. Express each of the following statements into logical operates, predicates and quantifiers.

(i) Let $F\,(x\,, y)$be the statements "$x$ can fool $y$," where the domain consists of all people in the world . Use quantifiers to express each of these statements.

(a) Everybody can fool fred.
(b) Evelyn can fool everybody.
(c) Everybody can fool somebody.
(d) There is no one who can fool everybody.
(e) Everyone can be fooled by somebody.
(f) No one can fool both Fred and Jerry.
(g) Nancy can fool exactly two people.
(h) There is exactly one person whom everybody can fool.
(i) No one can fool himself or herself.

(j) There is someone who can fool exactly one person besides himself or herself.

(ii) Use quantifiers and predicates with more than one variable to express these statements.

(a) Here is a student in this class who can speak Hindi.
(b) Every student in this class plays some sport.
(c) Some students in this class has visited Alaska but has not visited Hawaii.
(d) All student in this class have learned at least one programming language.
(e) There is a student in this class who has taken every course offered by one of the Departments in this school.
(f) Some students in this class grew up in the same town as exactly one other student in this class.
(g) Every student in this class has chatted with at least one other student in at least one chat group.

3. (i) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.
(a) $\forall x(x^2 \geq x)$ (b)-$\forall x(x > 0 \vee x < 0)$ (c) $\forall x(x = 1)$

(ii) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.
(a) $\forall x(x^2 \neq x)$ (b) $\forall x(x^2 \neq 2)$ (c) $\forall x(|x| > 0)$

(iii) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.
(a) $\forall x \forall y(x^2 = y^2 \rightarrow x = y)$ (b) $\forall x \exists y(y^2 = x)$ (c) $\forall x \forall y(xy \geq x)$

4. (i) Translate these statement into English, where the domain for each variable consists of all real numbers.
(a) $\forall x \exists y(x < y)$ (b) $\forall x \forall y(((x \geq 0) \wedge (y \geq 0)) \rightarrow (xy \geq 0))$ (c) $\forall x \forall y \exists z(xy = z)$

(ii) Translate these statements into English, where the domain for each variable consists of all real numbers.
(a) $\exists x \forall y(xy = y)$ (b) $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$
(c) $\forall x \forall y \exists z(x = y + z)$

(iii) Let $Q(x, y)$ be the statement "$x$ has sent an e-mail message to $y$," where the domain for both x and y consists of all students in your class. Express each of these quantification in English.
(a) $\exists x \exists y Q(x, y)$ (b) $\exists x \forall y Q(x, y)$ (c) $\forall x \exists y Q(x, y)$
(d) $\exists y \forall x Q(x, y)$ (e) $\forall y \exists x Q(x, y)$ (f) $\forall x \forall y Q(x, y)$

(iv) Let $P(x, y)$be the statements "student x has taken clas y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school.
Express each of these quantifications in English.

(a) $\exists x \exists y P(x, y)$ (b) $\exists x \forall y P(x, y)$ (c) $\forall x \exists y P(x, y)$
(d) $\exists y \forall x P(x, y)$ (e) $\forall y \exists x P(x, y)$ (e) $\forall x \forall y P(x, y)$

5. Express each of the following statements using predicates, quantifiers, logical connectives and mathematical operators.
   (a) The product of two negative integers is positive.
   (b) The average of two positive integers is positive.
   (c) The difference of two negative inters need not be negative.
   (d) The difference of a real number and itself is zero.

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**Practical 3**

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## Rules of Inference and Methods of Proof

1. Use rules of inference to show that the hypotheses "Randy works hard", "if Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

2. Test the validity of the following arguments by using the laws of logic.
   a. Team A will win the cricket match if they are playing against team B. If team A does not win then team C will take away the trophy. Team C does not get the trophy. Hence team A does not play against team B.
   b. If it rains then I carry an umbrella. If it shines then I do not need a sweater. Either it rains or it shines. I do need a sweater. Hence I carry an umbrella.
   c. The book is readable iff the print is clear. Either the print is clear or printer is bad. The printer is not bad. Hence the book is not readable.
   d. I like meeting people. I like travelling also. If I like meeting people and travelling then I am considered to be a nice person. Hence I am a nice person.
   e. If I study, I learn. If I don't study, I have a good time. Therefore either I learn or I have a good time.
   f. By preparing truth table test the validity of the following argument:
      Sudhir is either clever or lucky. Sudhir is not lucky. If Sudhir is lucky then he will win the lottery. Therefore, Sudhir is clever.
   g. If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in economics Today is Tuesday and my Economics professor is sick. Therefore I have a test in Mathematics.

3. Using direct proof prove that if $m+n$ and $n+p$ are even integer, where m, n and p are integers, then $m+p$ is even.

4. Show that if n an integer and $n^3+5$ is odd, then n is even using
   a) a proof by contrapositive. b) a proof by contradiction.

5. Prove that if n is a positive integer, then n is odd if $5n+6$ is odd.

6. Show that every odd integer is the difference of two squares by using method of direct proof.

7. If $n$ is an integer and $3n + 2$ is even then prove that n is even by using proof by contrapositive.

8. Prove or disprove (by counter example)
   a. The sum of three consecutive even integers is divisible by 6.
   b. The sum of three consecutive odd integers id divisible by 6.

9. Show that the statement "Every positive integer is the sum of the squares of two integers" is false.

10. Prove that every odd integer is the difference of the squares of two integers.

**Practical 4**

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## COUNTING I

1. The English alphabets contain 21 consonants and 5 vowels. How many strings of six lowercase letters of the English alphabets contain

   a) Exactly one vowel b) Exactly two vowels

   c)At least one vowel d) At least two vowels **(Oct-2018, 8 Marks)**

2. How many functions are there from the set $\{1, 2, 3, \dots, n\}$ where $n$ is a positive integer to the set $\{0, 1\}$

   a) that are one-to-one b) that assigns 0 to both 1 and $n$

   c) that assigns 1 to exactly one of the positive integers less than $n$

   **(Oct-2018, 8 Marks)**

3. Show that at least ten of any 64 days chosen must fall on the same day of the week. **(Oct-2018, 4 Marks)**

4. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? **(Oct-2018, 4 Marks)**

5. How many ways can we form a committee of three person from a set of ten men and eight women such that the committee consists at least one women?

   **(Apr-2018, 8 Marks)**

6. In how many ways can a photographer at a wdding arrange six people in a row including the bride and groom if

   a) the bride must be next to groom b) the bride is not next to the groom

   c) the bride is positioned somewhere to the left of the groom

   **(Apr-2016, Oct-2017, 8 Marks)**

7. How many bit strings are there of length six or less not counting the empty string? **(Oct-2015, 8 Marks)**

**Practical 5**

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**COUNTING II**

1. How many ways are there to distribute six indistinguishable balls into nine distinguishable bins? **(Oct-2018, 4 Marks)**

2. Suppose that a large family has 14 children including 2 sets of identical triples, 3 sets of identical twins and 2 individual children. How many ways are there to seat children in a row of chairs if the identical triplets cannot be distinguished from one another? **(Oct-2018, 4 Marks)**

3. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$ where $x_i, i = 1, 2, 3, 4, 5$ is a non-negative integer such that
   a) $x_1 \geq 1$ b) $x_i \geq 2 \ for \ i = 1, 2, 3, 4, 5$ **(Oct-2018, 8 Marks)**

4. How many ways are there to arrange the letters in the word "MATHEMATICS"? **(Oct-2017, 4 Marks)**

5. How many solutions does the equation $x_1 + x_2 + x_3 = 11$ where $x_i, i = 1, 2, 3$ is a non-negative integer such that

   a) $x_1 > 1, x_2 > 2, x_3 > 3$ ? b) $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

   **(Oct-2017, 8 Marks)**

6. How many ways are there to distribute six different toys to three different children such that each child gets at least one toy?

   **(Oct-2015, 4 Marks)**

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**Practical 6**

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**The Principle of Inclusion and Exclusion**

1. Write Inclusion-Exclusion principle for three sets and four sets.

2. Find the number of elements in $A \cup B \cup C$ if there are 100 elements in each set and if
   i. The sets are pairwise disjoint
   ii. There are 50 common elements in each pair of sets and no element common in all three sets
   iii. There are 50 common elements in each pair of sets and 25 elements common in all three sets.
   iv. The sets are equal.
3. If $S_1, S_2, \dots \dots \dots, S_n$ are non-empty sets then prove that
$$|S_1 \times S_2 \times \dots \times S_n| = |S_1| \cdot |S_2| \cdot \dots \cdot |S_n|$$

4. If $S_1, S_2, \dots \dots \dots, S_n$ are pairwise disjoint subsets then prove that
$$|S_1 \cup S_2 \cup \dots \cup S_n| = |S_1| + |S_2| + \cdots + |S_n|$$

5. Out of 100 cups of coffee 54 are too hot, 32 are too cold, 28 are too bitter, 17 are too hot and too bitter, 9 are too cold and too bitter. Find out how many cups are just right.

6. In a group of 200 students, 80 are taking Mathematics, 60 are taking chemistry and 30 are taking both subjects. How many students are taking either Mathematics or Chemistry? How many students are taking neither subjects?

7. There are 280 colleges affiliated to university, each having at least one of the three facilities viz. hostel, credit shop and career guidance facility. The data reveals as '250 colleges have hostel, 80 have credit shops and 40 have career guidance facility'. Also 30 colleges have all the three facilities. Find how many colleges have exactly two facilities.

8. In a survey, 2000 people were asked whether they read India Today or Business Times. It was found that 1200 read India Today, 900 read Business Times and 400 read both. Find how many read at least one magazine and how many read neither.

9. In a computer laboratory out of 6 computers, 2 have arithmetic unit, 5 have magnetic disk memory, 3 have graphics display, 2 have both arithmetic unit and magnetic disk memory, 3 have both magnetic disk memory and graphic display, 1 has both arithmetic unit and graphic display, 1 has arithmetic unit, magnetic disk memory and graphic display. How many have at least one specification?

10. How many integers between 1 and 500 are divisible by either 2 or 3 or 5?

11. How many are there in the union of four sets if the sets have 50, 60, 70, and 80 elements respectively. Each pair of the sets has 5 elements in common, each triple of the sets has 2 elements in common and 1 element is common in all four sets?